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Effect of specimen shape on the behavior of brittle materials using probabilistic and deterministic methods

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Abstract

The main objective of this paper is to verify the activated defect localization under various loading rates for Modified Brazilian Disk (MBD) compared with spherical glass specimen frequently used in the literature. The geometry of specimen can considerably influence the mechanical response of material specially, the brittle materials which are very sensitive to the defects activations and finally corresponding Weibull statistic problematic for these materials. The high and low loading rates have been investigated using universal Instron machine compression and Compressive Hopkinson Pressures Bars (CHPB) on MBD lead glass specimen and then the experimental results have been compared via the Weibull's distribution for scatter strength variation. Finally, we observed the contact problem leading to the activated defects in this area on spherical glass specimen under static and dynamic loading not MBD specimen under both loading rates. Consequently, the probabilistic approach should be taken into account including the specimen geometry and also 'Type' of loading application due to the activated defect position arising from the sever contact problem for the brittle materials.

Keywords: Brittle materials; Defect localization; Weibull distribution

1. Introduction

The brittle materials, especially the glass materials, show the perfect elastic behavior until the failure occurs and present a substantial scatter resistance value under applied loadings contrary to the ductile ones. The absence of crystal network and pre-existing micro defects provide the sensitivity concept based on stress concentration around the defects.[1] It shows less resistance value than the theoretical atomic cohesion and also unpredictable strength value. Hence, a randomly defect distribution and its corresponding activation mechanism by applied stress can determine the resistance of the brittle materials and its intrinsic mechanical characteristic. Because of those reasons, the probabilistic approach as Weibull's distribution including two parameters has become a practical method to assess the variation of resistance for the brittle materials.[2,3] According to the Freudental's proposition,[3] the probability of occurrence of a critical defect decreases at small volume while it can increase at the large volume as follows:

$$F(V) = 1 - \exp\left[-\left(\frac{V}{V_0}\right)\right] \quad (1)$$

where V_0 is the mean volume occupied by a defect, V is the volume and $F(V)$ is the probability of an occurrence of critical defect in the volume. The Griffith's experimental tests of strength of material for glass fiber diameter effect can be explained by the above relationship (Eq. (1)). Due to the aforementioned probabilistic idea, the concept of volume and the consideration of the quantity of defect have also become as an essential point of view to consider the probabilistic approach. Considering Eq. (1), Weibull proposed the probability of failure replacing the applied stress in the volume occupied by a defect:

$$P_r(\sigma) = 1 - \exp\left(-V\left(\frac{\sigma}{\sigma_0}\right)^{\beta}\right) \quad (2)$$

where $P_r(\sigma)$ is the failure probability, σ_0 is called a scale parameter which is proportional to the mean stress and β is Weibull's modulus which deals with the divergence in outcomes. The volume size effect on probability of failure in the Weibull's

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distribution should be taken into account. The above relationship indicates a two-parameter of the Weibull's distribution, which is frequently used. Many studies show that the brittle materials, especially glass materials present more random strength for which the Weibull's modulus β is about 5[4] than the ceramic's ones for which the β is placed between 10 and 20.[5] The Weibull's modulus for dynamic loading application is frequently utilized as well. Recently, the study concerning the strength of the brittle materials under high loading rate has become an important subject regarding to several industrial applications as foreign object impact and thermal shock problem. It is well known that the brittle materials exhibit the strength increase under high loading applications.[6–12] A progressed research was carried out to demonstrate more strength variation sensitivity reason under the high loading application for the brittle materials than the ductile materials via Weibull's modulus.[9] The dynamic fracture on the brittle materials leads to the particular failure mechanism like a multi-activation of the defects and the failure of the whole of material not simple fracture from a critical defect. These fracture mechanisms have been interpreted as a damage phenomenon in the literature.[4,11] Some works show that the mentioned dynamic brittle fracture mechanism provides less scatter value than the static's one and so, the deterministic approach was proposed in Ref.[6] According to their works, the brittle materials exhibits "one" strength value if it is subjected to the high loading rate. However, other studies indicate that the scatter value for high velocity loading is greater than the static ones for the brittle materials.[7,8] Considering these controversial results which have been reported in the literature, which include probabilistic and/or deterministic approaches according to the various loading rates, in this paper it is attempted to clarify and simplify this diverge phenomenon by means of the fracture pattern mechanism analysis due to the activated defects position, contact problem on the specimen geometry and the loading type on the brittle materials. For these reasons, the application of the quasi-static and high loading rates on lead glass material is performed and the experimental results are analyzed via the Weibull's distribution and localization of the activated defects on the specimen geometry and loading type. In the next section, the utilization of Coefficient of Variation (CV) will be presented to show its validation in practical application. It is able to simplify the complicate probability function and also allows us to investigate and explore the experimental results of the other studies in literature herein.

2. Weibull's modulus β and its simplified form of "Coefficient of Variation (CV)[13]"

The Weibull's modulus β of Eq. (2) for this distribution can be extracted using simple rank regression analysis from the density function as follows:

$$\mathrm{Ln}\left[\mathrm{Ln}\left(\frac{1}{1-P_\mathrm{r}}\right)\right] = \beta\mathrm{Ln}(\sigma) + \beta\mathrm{Ln}\left(\frac{1}{\beta}\right) \tag{3}$$

and

$$P_\mathrm{r} = \frac{i}{n+1} \tag{4}$$

where i is the number of specimens and n is the amount of specimen. The method of Coefficient of Variation (CV) for the statistical distribution of material properties including the ultimate strength is a very feasible statistical way. The CV method can be defined as the ratio of the standard deviation to the mean value as below:

$$\mathrm{CV} = \frac{\sigma_\mathrm{s}}{\mu_\mathrm{s}} \tag{5}$$

where σ_s and μ_s are standard deviation and mean values, respectively. The complicated function may be transformed into a simple form by using the concept of Coefficient of Variation. It greatly simplifies the calculation of statistical parameters and the obtained results are very close to the original functional relationship. The CV can be determined for the most common distribution as exponential, log-normal and Weibull's distribution. In this paper, two parameters Weibull's distribution and CV concept are employed. The probability of Weibull's density function can be written as:

$$f(\sigma) = \frac{\beta}{\sigma_0}\left(\frac{\sigma}{\sigma_0}\right)^{\beta-1} \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^{\beta}\right] \tag{6}$$

From the definition of the mean and the standard deviation values for each probability density function, we can deduce the mean and the standard deviation value of the Weibull's distribution as follows:

$$\mu_\mathrm{s} = \sigma_0 \Gamma\left(1+\frac{1}{\beta}\right) \tag{7}$$

$$\sigma_\mathrm{s} = \sigma_0 \left\{\Gamma\left(1+\frac{2}{\beta}\right) - \left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\} \tag{8}$$

$$\mathrm{CV} = \left[\frac{\Gamma(1+(2/\beta))}{\Gamma(1+(1/\beta))^2} - 1\right]^{0.5} \approx \beta^{-0.93}$$
$$\text{for} \quad 1 \le \beta \le 50 \tag{9}$$

where $\Gamma$is gamma function. The above relationship Eq. (9) allows evaluating the Weibull's modulus value using the Coefficient of Variation.

3. Quasi-static strength of the glass material depending shape specimen

Firstly, the quasi-static case is considered. The experiments on lead glass material disk under lateral compression are performed to investigate the position of crack initiation and the scatter in maximum failure loads based on the Weibull's distribution. The obtained values will be compared with the ones from a spherical glass specimen under compressive loading.

3.1. Experiments

The uni-axial compression Brazilian test is commonly used on the brittle materials to apply the compression at top of the disk specimen. This loading condition can provide the indirect tensile strength in direction of the median plane. In the experiment,

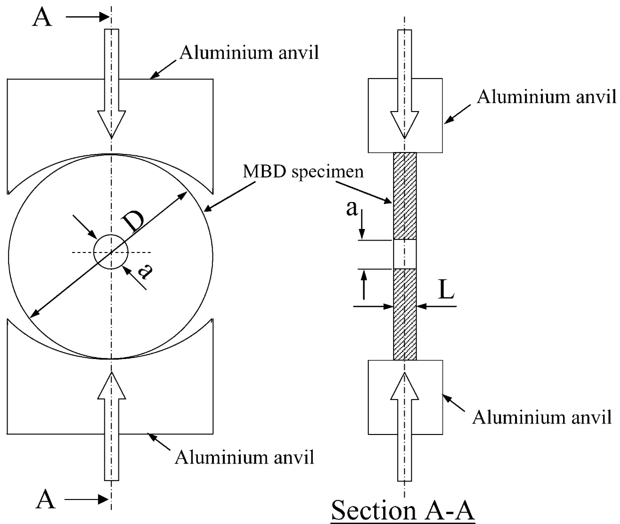

Fig. 1. Modified Brazilian Disk specimen configuration including lateral compression test setup; $D = 20$ mm, $a = 2$ mm, $L = 6$ mm.

the Brazilian Disk consists of one small central hole ($a = 2$ mm diameter) on the outer disk of which diameter and thickness are 20 mm (=D) and 6 mm (=L), respectively (Fig. 1). That geometry is taken to induce the stress concentration around the notch tip. Hence, this Modified Brazilian Disk (MBD) lead glass material will be broken using the notch tip crack concept. All specimens have been manufactured by the molding technique of injecting the hot crystal liquid (more than 800 °C) in prepared shape. This method makes it possible to avoid some undesirable defects on the surface of specimen during manufacturing process. The applied lead glass specimen in the experiment has the mechanical properties in Table 1.

The compressed MBD lead glass is placed between two semi-cylinder aluminum anvils having adjustable specimen form (Fig. 1). This configuration of anvils prevents the maximum pressure presence at the contact point during the compression application process. The compression tests were performed using Instron 4505 servo-hydraulic testing machine. The stroke velocity was controlled to have 10^{-5} mm displacement per minute. When a failure occurs, the load is instantaneously reduced and the test is stopped. The ultimate strength corresponds to the maximum load recorded during the test.

3.2. Tensile strength of MBD lead glass and probabilistic analysis

When the failure occurs, the ultimate load is recorded and the maximum stress can be calculated:

$$\sigma_r = \frac{2K_t P}{LD\pi} \tag{10}$$

Table 1
Mechanical properties of applied lead glass material

Young's modulus (E)	59 GPa
Poisson's ratio (ν)	0.218
Density (ρ)	4350 kg/m^3

Table 2
Statistical tensile strength results for MBD glass materials

Tensile strength (MPa)	Mean value (MPa)	Standard deviation (MPa)
Lead glass	46.70	8.60
Soda-lime glass[11]	96.30	13.66

where σ_r, K_t, P, L and D are the maximum stress, stress concentration factor, failure load, thickness and diameter, respectively.[14] The stress concentration factor for illustrated MBD specimen (Fig. 1) is equal to 2. The main result is summarized and compared with the soda-lime glass of which test is performed on the same MBD specimen and the same configuration (Table 2).

The static strength experimental outcomes are displayed using rank regression analysis to determine the Weibull's modulus β (Fig. 2). The value of Weibull's modulus β can be also calculated by means of Coefficient of Variation (CV, Eqs. (5) and (9)) and the value of 6.1679 is obtained. This value demonstrates a very good approximation with numerical results including 7% mistake. It is due to the coefficient of correlation value. Therefore, the utilization of CV can be usually variable on the brittle materials if the mean and the standard deviation values are known.

3.3. Fracture mechanism analysis based on the finite elements method

Many researches have been carried out several compression tests on the spherical glass specimen to determinate the defects initiation and the strength of material. In the experiment of this paper, the MBD commercial lead glass specimen was carried out and this specimen geometry allows us to compare the role of the defects on the surface under inducing indirect and compressive stress with spherical glass specimen form in the literature that failure occurred on the surface contact zone[15] (Fig. 3a and b). A photograph of the fragmentation after test is shown in Fig. 3a. In general, all MBD lead glass specimens are split into six pieces

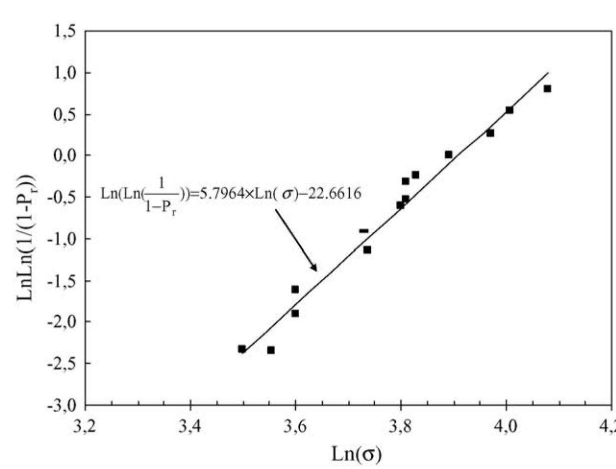

Fig. 2. Rank regression analysis for MBD lead glass: the line corresponds to the Eq. (4) and tangent, β represents the scatter value (5.7964) including correlation coefficient ($R^2 = 0.975$).

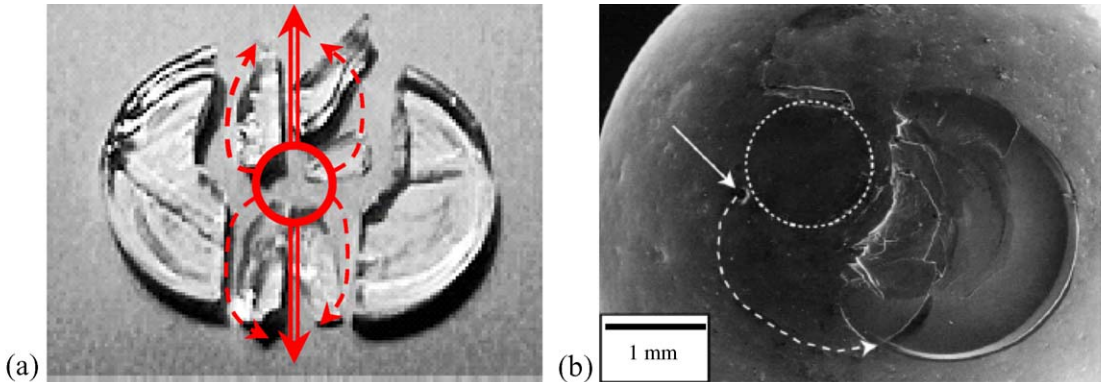

Fig. 3. (a) General mechanism of fragmentation for MBD lead glass specimen subjected to the lateral compression including quasi-static loading and (b) fracture start localization on the soda-lime glass sphere specimen[15].

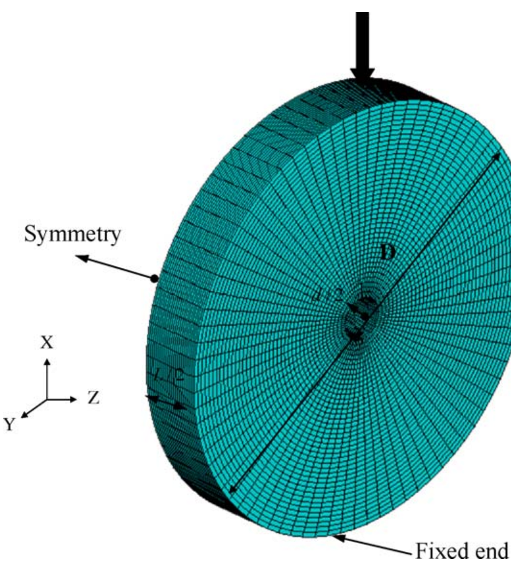

Fig. 4. Mesh configuration and corresponding boundary conditions.

at the small hole (ring appeared in Fig. 3a) and the cracks very rapidly propagate (not visible by naked eye) toward both direction of top and bottom of the specimen (straight and dashed lines in Fig. 3a). The fragmentation provokes crash failure.

This mechanism of fragmentation is entirely different and compared with the spherical glass specimen which is comminuted and reduced into very thin powder. The stress distribution of MBD specimen and high stress concentration which might be a cause of failure near the central hole are needed to be investigated via numerical simulation methods and corresponding results provide more comprehensive understanding of the macro level fracture mechanism view. The mesh is made only in a half of MBD specimen because of the symmetric assumptions. The bottom of geometry is completely fixed and the concentrated force equivalent to failure's one is applied at the top of specimen (Fig. 4).

The simulation result using the finite element implies that the indirect tensile stress or opening stress is distributed on the vertical direction toward the applied static loading and the stress in *y*-direction (σ_{yy}) is located near the center of hole which attains

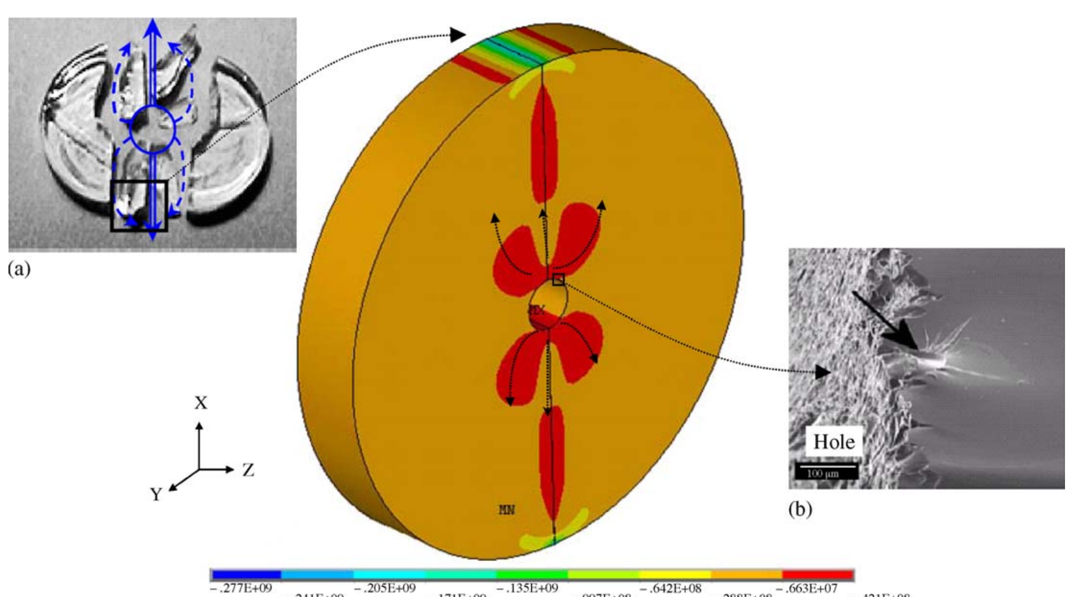

Fig. 5. (a) One half symmetric FEM model indicating stress in *y*-direction (in Pa); (b) localisation of the fracture start point by SEM.

more than 41 MPa under an average compressive failure force (1.25 kN) (Fig. 5).

Consequently, the cracks propagate on the vertical direction from the central hole. This primarily results in a fracture of specimen in median plane. The second crack is placed in 45° direction (diagonal direction) and it propagates toward the top of specimen and creates a second fracture. The FEM results make it feasible to explain the mentioned mechanism of fracture in MBD specimen (Fig. 3a). The high compressive stress near bottom and top of MBD specimen can be observed. The aforementioned positions represent the compressive stress more than 260 MPa based on FEM results. The FEM analysis agrees with Hertzian cone crack (ring and dashed line in Fig. 5a) morphology. In spite of the fact that the contact zones are locally damaged during the quasi-static test process, the emphasized zones never change the failure mechanism and mechanical resistance due to critical defect inactivation. Moreover, it is attempted to find out the origin of fracture for MBD specimen by means of scanning electric microscope (SEM) around the small center hole at which the maximum tensile stress happens (Fig. 5b). The failure mechanism of MBD specimen is very different compared with the spherical glass specimen which is broken into the fine powder. In fact, the failure occurs near the maximum tensile stress point and it is possible to detect a small piece which is lost due to the contact pressure problem just under the point of loading application. This point does not cause the specimen failure. The similar compressive test for MBD soda-lime glass specimen has been carried out with different thickness for 5 mm diameter.[11] The experiment using such a configuration of specimen showed that more fragile fractures happened with greater diameters. This effect of the strength decrease due to larger specimen size has been observed by several authors[16,17] and it is also predicted by certain statistical approaches (Eqs. (1) and (2)).

4. Discussion

Brajer et al.[15] performed the compressive test on the spherical glass specimen with various diameters from 2 to 8 mm in order to obtain the relationship between the maximum loading and different diameter. The spherical glass specimen has been compressed between two steel planar anvils and aluminum shim insertion. They have measured the variable strength including standard deviation and means values of spherical glass specimen according to the increasing volume but not the Weibull's modulus value β. Firstly, their results are shown as solid line in Fig. 6, where it can be observed that larger sphere diameters tend to have greater mean load value. This is contrary to the expressed relations in Eqs. (1) and (2) (Freudental-Weibull's failure probability approach) which present asymptotic variation of failure probability versus volume augmentation. Furthermore, the aluminum shim insertion under the steel plate anvil considerably increases the strength. However, there was no explanation for the reason of the above results.

The validation of CV, which is a practical method, is checked in Section 3.2. Using the CV, the Weibull's modulus value β can be calculated in this paper and also presented as two dashed lines in Fig. 6. Moreover, the parameters, β, critical stress and affected

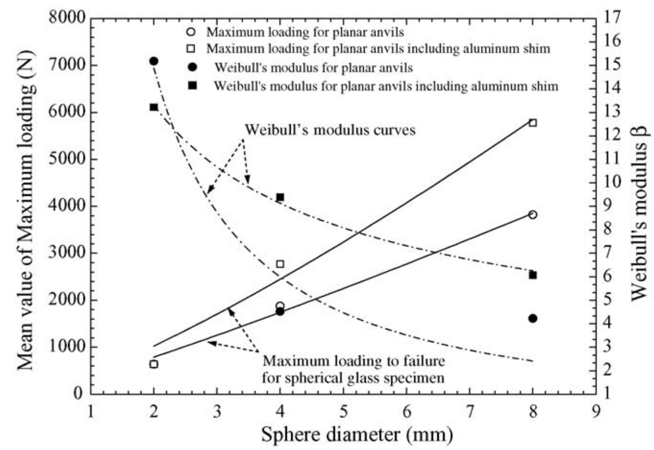

Fig. 6. Mean failure loading values and Weibull's modulus as a function of diameters of spherical glass specimen with and without aluminum shim insertion.

volume are more sensitive without aluminum shim insertion. This means that added aluminum shim at the bottom and top of tested specimen can reduce the contact problem severity. It is also found that the failure takes place just under the contact surface (Fig. 3b). The evaluation demonstrates that the spherical glass specimens of small diameter provide less random behavior ($\beta > 15$) and low mechanical resistance. To understand and resolve these controversial results, we investigate the contact problem nature in considered spherical specimens which have been commonly ignored.

4.1. Analysis of contact problem on spherical specimen under compressive loading

In Fig. 7, the selected spherical glass specimens with different diameters including contact considerations are presented for the identical loadings (880 N). The results of finite element method in median plane are focused on using 8-node structural brick element of ABAQUS code. The tensile stress and compressive stress are exhibited under contact condition. The comparison between compressive and tensile zones shows that the contact weight has influence on mechanical resistance of chosen specimens. As a matter of fact, low spherical glass specimen diameter yields severe contact effects and the current effects disturb mechanical resistance nature, i.e. the mechanical resistance of material explicitly depends on induced contact state in specimen. In fact, the spherical experiments are not an appropriate test method to evaluate the mechanical resistance of the brittle materials due to highly contact-dependent intrinsic characteristics.

According to the experimental results reported in Ref.[15], our evaluation for β value and FEM results (Fig. 7) indicates that the contact area is increased with the sphere diameter and aluminum shim insertion of which Young's modulus value is smaller than steel's one. Consequently, the pressure amplitude on contact area decreases, i.e. the contact pressure has an effect on the real strength of material and its corresponding intrinsic parameter as Weibull's modulus β. From the above facts, the following information can be drawn in this paper:

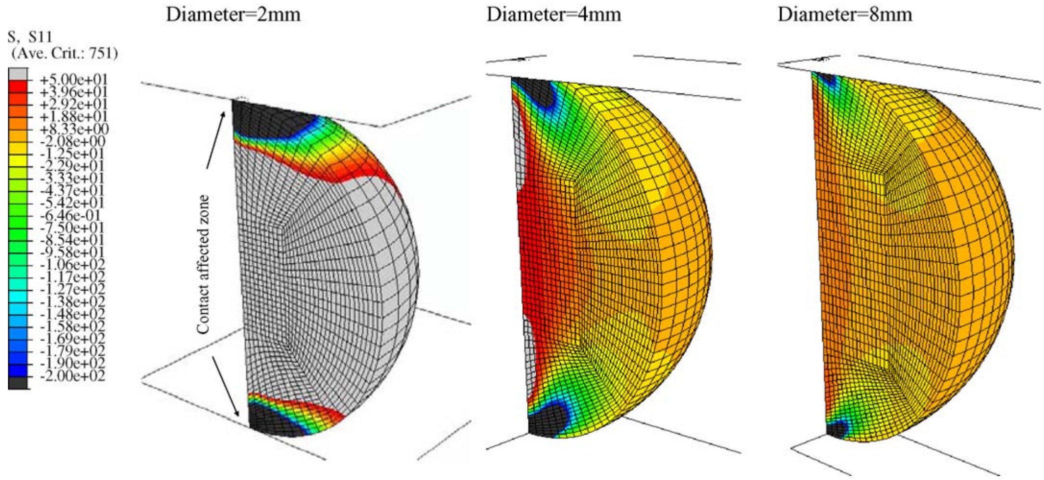

Fig. 7. Contact severity variations for spherical glass specimen with different diameters using stress distribution in loading direction (in MPa) including identical axial force (880 N) and one-fourth symmetry assumption.

- the geometry of specimen can influence both the strength of material and the scatter value.
- the condition of loading and the relative contact problem can manipulate the strength of the brittle materials.

Furthermore, the activated defects are not localized near the contact area at the maximum tensile stress point for MBD specimen. Nonetheless, different mechanical behaviors have been observed for the spherical glass specimen in which the fracture always occurs near the contact zone.[15] This inducing contact problem due to the loading type condition and the specimen geometry can be verified for the dynamic loading application as 'edge-on-impact' or ballistic test. In the next section, an experiment of the dynamic loading application via Compressive Split Hopkinson Pressure Bars (CSHPB) for MBD lead glass specimen is explained and its result is compared with the edge-on-impact reported in the literature.

5. Dynamic loading via Compressive Split Hopkinson Pressure Bars (CSHPB)

In the previous section, it was demonstrated that the crack initiation is located adjacent to the central hole, where the maximum tensile stress is applied to MBD specimen, instead of the contact zone. A similar experiment for dynamic condition via CSHPB is presented in this section. Additionally, the cracking pattern will be examined under this condition test and compared with the edge on impact or ballistic test in which the contact pressure zone is still available as the spherical glass specimen under static loading.

5.1. Experiments and results

It is well known that the strength of the brittle materials can increase up to two times greater than the static's one under the dynamic loading condition. In addition, this high loading rate yields a damage phenomenon in these materials inducing the different crack pattern relative to the static's one. The Compressive Split Pressure Hopkinson Bars test is employed to apply a dynamic loading to MBD specimen. As a matter of fact, the MBD specimen is placed between two bars which are called Incident and Output bars whose lengths are 1100 and 900 mm, respectively (Fig. 8). A striker bar of 500 mm is launched to apply a perfect impact to the incident bar. Two mounted gauges on Output and Incident bars are used to record the electric signals which represent the compressive strain and tensile strength of the specimen, respectively. The mismatch of acoustic impedance between the metal bars and the lead glass specimen causes that the elastic compressive longitudinal wave partially transmits into the specimen and arrived on the Output bar. Assuming non dispersive one dimensional wave propagation theory, the applied velocity of deformation corresponds to 275/s in this paper. The obtained stress amplitude due to this applied velocity of deformation assures the dynamic equilibrium theoretical approach that the dynamic fracture time of material must not be less than three times of celerity in this material.[18]

It has been found that the strength of lead glass material under the dynamic loading instead of static's one increases from 46.7 to 93 MPa. The various scatter levels for two loading conditions are compared using Weibull's distribution in Fig. 9. The Weibull's modulus has been calculated ($\beta = 3.60$) and this value is smaller than the static's one ($\beta = 5.79$). The above values have the same range as soda-lime glass whose range is mostly about 5.

Fig. 10 shows a typical dynamic fracture result of MBD lead glass by CSHP and dynamic fracture of glass material by 'edge-on-impact' test. In Table 3, there are the measured mean maximum strength and standard deviation values. The experiments demonstrate that the MBD lead glass specimen was broken into several fragmentations on regular fracture pattern without high

Table 3
Statistical results of MBD Lead glass tensile strength

Tensile strength (MPa)	Mean value (MPa)	Standard deviation (MPa)
Dynamic test	93.0	21.0

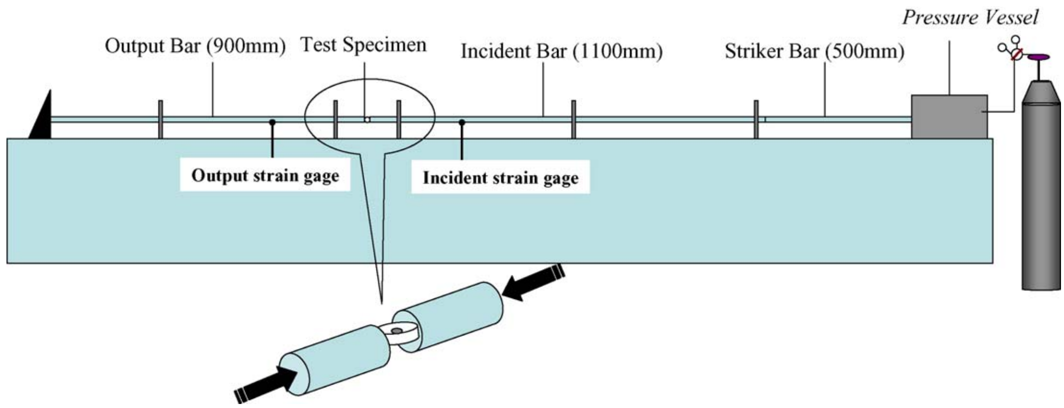

Fig. 8. Compressive Split Hopkinson Pressure bars general configuration.

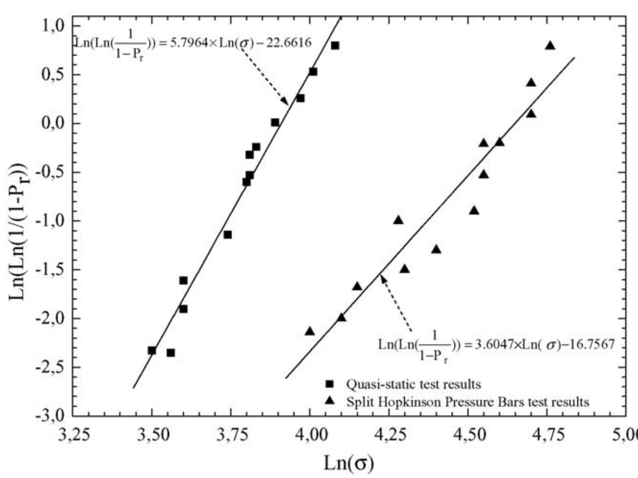

Fig. 9. Weibull's modulus presentation including the static and dynamic loadings.

compressive zone which can be found for the contact problem (Fig. 5a).

The current dynamic fracture can be elucidated by the simultaneous activation of majority of flaws by compressive pulse stress propagation in specimen body. The fracture mechanism results lead to the augmentation of the strength of material under dynamic loading based on the energy intensity equality described by the area of applied stress and fracture time.[4] Consequently, the fragmentation including damage is proportional to the applied energy intensity. It is supposed that the above fragmentation pattern of MBD specimen can be initiated on a large amount of fragments to be detached along the tensile stress concentration which can be obtained by finite element method (Fig. 5) during a very short time (the average of failure time is about 50 μs for all specimens). Fig. 11 illustrates steps of the assumed fragmentation. Firstly, the compressive wave propagation provokes the activation of a large number of micro-crack in the meridian plane (Fig. 11a) and then, the diagonal direction (Fig. 11b). Finally, the micro-cracks appear in the whole specimen (Fig. 11c) and failure happens. Consequently, the fracture pattern is regular around the small hole and propagates on the whole material. The contact Hertzian problem in these failure types cannot be detected and the fracture is due to the defects not on a surface but in volume entity.

5.2. Discussions

Our experimental results show that the dynamic failure mechanism by regular crack pattern in volume results in a damage phenomenon as fragmentation and also more random strength value. The edge-on-impact test or ballistic test represents the same damage mechanism that can be detected via CSHPB condition. Fig. 12 presents an impact zone (dark zone) by a lead bullet (Fig. 12a) and a steel one (Fig. 12b) on soda-lime glass subjected

(a) (b)

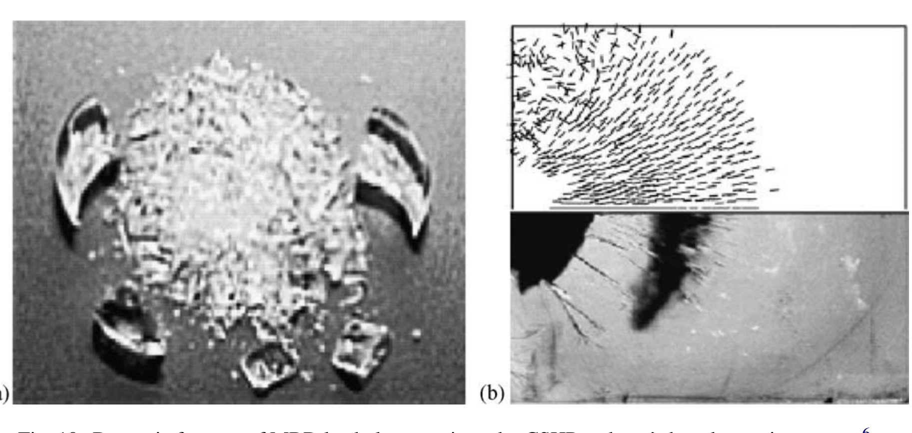

Fig. 10. Dynamic fracture of MBD lead glass specimen by CSHP and one's by edge-on impact test[6].

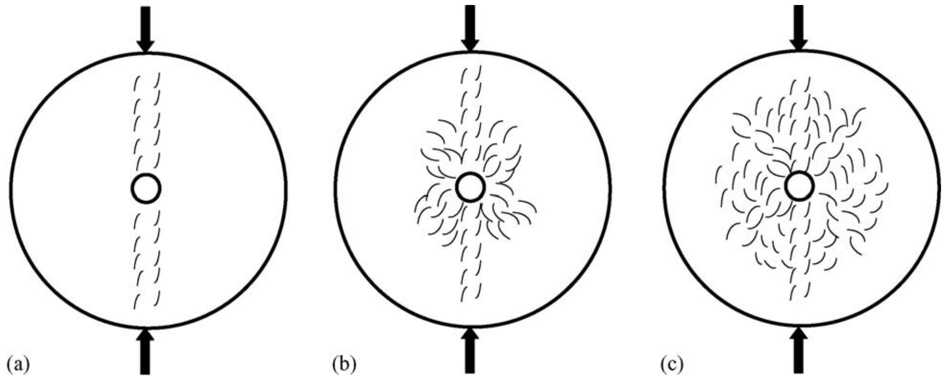

Fig. 11. Schematics of the main configuration of failure for MBD specimen at high velocity impact.

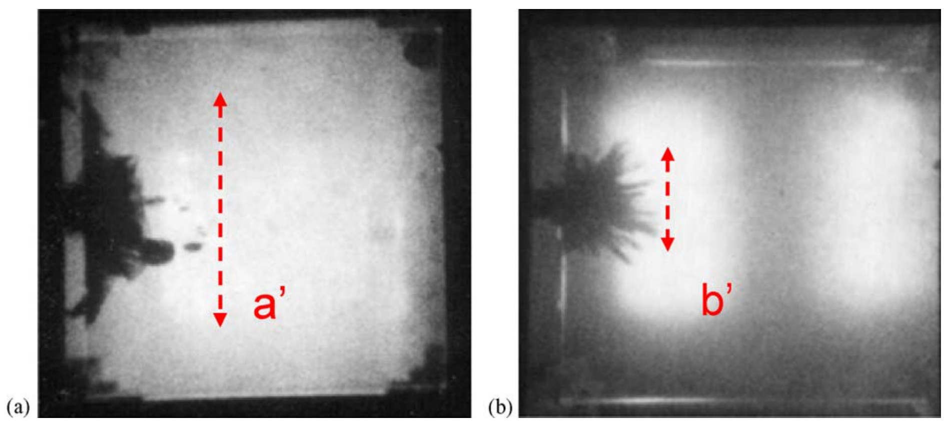

Fig. 12. (a) Cracking pattern 10 μs after impact of glass by soft bullet and (b) hard bullet [10].

to the ballistic test.[15] From the results reported in Ref.[15], the dark zones correspond to the damage ones and this zone in the vicinity of the impact is totally comminuted. The second area illustrates a high density of radial and orthoradial cracks and the propagation of long radial cracks appears in the third zone.

The above fracture mechanism is a typical fracture pattern for all applied specimens under impact loading. The obtained damage severity makes more important fragmentation for the soda-lime glass material subjected to the impact condition and it depends on the velocity of impact.[4,6,15] Hence, Brajer et al.[15] indicated that the soft bullet (lead bullet) provokes a large damage zone (Fig. 12a) compared with the hard bullet (Fig. 12b).

It should be recalled that the soft bullet was launched with less velocity (430 m/s) than the hard bullet (steel bullet, 820 m/s). The authors observed a Rayleigh's surface wave on the contact zone by the soft bullet and concluded that the activated defects under the impact are positioned on the surface of specimen not in volume like the spherical glass specimen under the static. In another work,[6] a deterministic approach for the mentioned condition loading was proposed to supersede the probabilistic one based on the obtained deterministic strength value under high loading rate. Accordingly, we conclude that the brittle materials, especially the glass material, subjected to impact condition should not be considered as a 'simple' dynamic problem due to the very high sensitivity contact zone and corresponding defect activations regarding to impact and CSHPB. For these reasons, the brittle material (MBD lead glass in this paper) shows greater scatter strength value, whereas under the impact loading condition, it can have less scatter strength value. It is also required to remind that the impact problem needs the deterministic approach.[6] Consequently, the application of dynamic loading on the brittle materials must be thoughtfully distinguished as the longitudinal elastic compressive wave propagation by means of CSHPB without contact zone and the same wave application with damaged contact zone by impact. Furthermore, the choice between the probabilistic approach and the deterministic one must be made based upon the dynamic loading type.

6. Conclusions

The compressive tests on MBD lead glass specimen under quasi-static and dynamic loadings have been performed. The static experimental results have been successively compared with the spherical glass specimen. According to the results, the failure initiation is located at the maximum tensile zone near the small hole to which the maximum tensile stress is applied, but not at the damaged contact zone, whereas the spherical glass specimen provokes failure by activated defects on the contact zone under the static condition. Hence, the contact pressure manipulates the behavior of the brittle materials due to the high defect sensitivities on surface. The MBD lead glass specimen is broken into multi-fragmentation under CSHPB and the compressive wave has provoked the defects not on surface but in volume. Therefore, the MBD lead glass specimen highlights greater strength and scatter data level compared with the static's one under dynamic loading. It is different from the results in the literature,[6,15] i.e. the activated defects located on damaged contact area via impact loading condition exhibit less scatter strength. Consequently, the contact problem has an effect on the scatter strength value under impact. Finally, the utilization of probabilistic method like Weibull's distribution is no longer valid if the materials behave under a rigorous contact problem.

7. Perspective

For the next work, it should be examined the validation of Weibull's distribution Eq. (2) on the brittle materials as:

$$P_s = f\left[\frac{V_d}{V}\right] \tag{11}$$

where P_s represents the severity of contact problem which depend on the ratio between the damaged volume by several condition test setup and the geometry specimen configuration V_d and the initial specimen volume V. The obtained numerical value P_s can be changed from 0 to 1; 0 is corresponding to the validation of Weibull's distribution and 1 for the no more valid probabilistic approach. The damaged volume V_d can be obtained by the finites elements method. The proposed relationship Eq. (11) can thereafter be integrated in the Weibull's distribution.

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